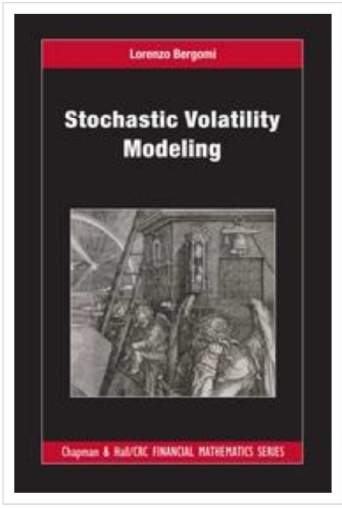
Stochastic Volatility Modeling

by Lorenzo Bergomi



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At a time when XVA Quants seem to be taking over the world of quantitative finance, more classical derivatives pricing is seeing its future called in doubt, and the question "Is Derivatives modelling dead?" is becoming a recurring topic at conferences and (mathematical finance related) social gatherings.

Lorenzo Bergomi, Quant of the Year 2009, and armed with 20 years' worth of quantitative modelling experience at Société Générale, brings here a clear answer: "NO!"

In fact, according to him, even though derivatives pricing has grown outstandingly over the past decades, the underlying risks are still largely misunderstood, and classical "calibration" has to be fully re-thought in terms of realised P&L of actual trading strategies.

This book should be read by practitioners, as it is the only one providing a strong quantitative framework to the (Delta and Vega) hedging of Equity derivatives. It should also be read by academics who will benefit from practical insights. It should finally be read by (motivated) students, who will definitely find areas to dig deeper in, both theoretically and numerically.

This monograph on stochastic volatility modelling is not a review of the state of the art literature on the topic. It maps the route to the new generation of models and how to trade with them. One of the goals here is to show that calibration of the implied volatility surface, though important, is far from enough, and first generation stochastic volatility models (such as Heston or local volatility models) are rarely able to go beyond this step. In fact, Lorenzo Bergomi honestly attempts to push these models further, but has to unfortunately acknowledge his (and other people who tried) failing: local and stochastic volatility models, as used until recently, are too structurally constrained by the initial volatility surface they try to match, and cannot effectively grasp the dynamics of forward variances, in particular when it comes to pricing cliquet options.

This is where Bergomi's contribution really kicks in. After a careful review of classical models, together with their hedging (in theory and in practice), he introduces and studies what I believe is his deepest contribution to the field: variance curve dynamics. Instead of suggesting dynamics for the instantaneous volatility, he proposed several years ago to model the forward variance curve directly. After motivating properly this new class of models, he shows how they naturally encompass classical stochastic volatility models (Heston in particular), and how they can be efficiently used to price options on realised variance and VIX Futures and options. Being infinite-dimensional objects, variance curves are a delicate beast to tame, especially numerically. To tackle this problem, he joined forces with then Société Générale collaborator Julien Guyon to develop an asymptotic expansion of prices assuming small volatility of volatility, and numerically validates it, thereby providing practitioners with easy-to-implement formulae. The final stretch of the book is to combine this variance curve approach with a local volatility model, in order to be able to capture both the current volatility surface, while having a framework flexible enough to capture various observed dynamics.

This book should be seen as a strong case for the need of a deeper understanding of derivatives' modelling (and their risks). Lorenzo Bergomi provides us here with new tools (variance curve models, metrics such as the At-The-Money Forward Skew and the Skew Stickiness Ratio) as well as new results on hedging and P&L computations of actual trading strategies, which have been so far too much overlooked in mathematical finance research. Welcome to the new era of Derivatives Modelling!