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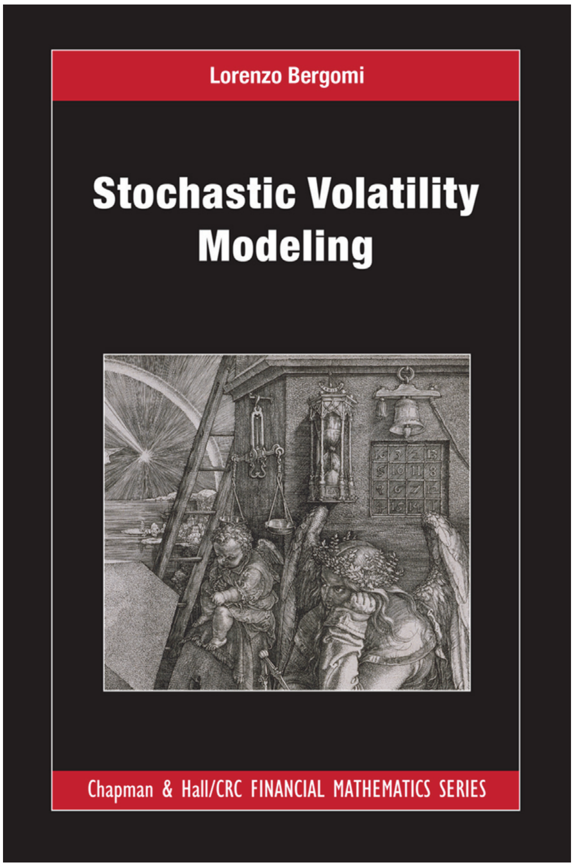
Book review



Stochastic Volatility Modeling, by Lorenzo Bergomi, Chapman Hall/CRC Press (2016). ISBN 978-1482244069.

In his book *Stochastic Volatility Modeling*, Lorenzo Bergomi organizes and shares the immense knowledge and experience on volatility modelling that he has accumulated over almost 20 years as head of quantitative research at Société Générale. As Société Générale has been a prominent market player in the area of equity derivatives, this makes the author particularly qualified to write about the subject. While the book, full of practical insights, is primarily intended for industry practitioners, it will also certainly be of great interest to academics. Indeed, Bergomi uses the book not only to gather in one single place—and further develop—his past publications, but also to introduce new, unpublished material. This inspiring new material addresses unsettled issues that certainly deserve further investigation and development; it actually may have the potential to open new lines of research that can keep academics busy for years.

Stochastic Volatility Modeling is a very original book. As the author warns in the preface, it is not a treatise nor a textbook. Bergomi's goal is to articulate the practical objectives of derivatives modelling, describe the thought process that led him to suggest and use particular models, and provide a thorough investigation of those. In fact, Bergomi approaches derivatives modelling like the physicist he used to be: it is about tackling a concrete problem, modelling it and producing some sort of a solution (price *and* hedge strategy) based on expansions and approximations that the traders can *use* to risk-manage a book of exotic options. He also writes about it like a physicist. For instance, one will not find the usual vocabulary of financial mathematics and stochastic processes—actually one will even struggle to find occurrences of the noun 'Itô' or the word 'martingale'. Mathematics-oriented readers may sometimes find that the exposition lacks rigour, wish that notions were defined more formally, that assumptions would be more clearly stated, and that more standard notation be used. But while they may have wished the exposition to be more formal, they will surely value Bergomi's great efforts of pedagogy in conveying and motivating practical insights and interesting ideas throughout the book.

Bergomi's starting point is that one hedges exotic options with the underlying asset *and* other options, usually vanilla options. As a consequence, the price and hedge depends on the joint dynamics of the hedging instruments: the underlying and vanilla options. However, it is notably difficult to consistently model the dynamics of an asset and the options written on this asset. Indeed, since all options are written on the same underlying security, there are hard relationships between option prices that the model must satisfy, e.g. call prices must be a decreasing and convex function of the strike. There also exists a hard relationship between an option price and the price of its underlying, since the terminal value of the option is a function of the underlying price. It is a formidable task and still an unsettled issue to derive tractable dynamics that satisfy those constraints. Chapter 4 describes a few interesting but failed attempts.

Since this path is impractical, Bergomi then suggests the following original approach. He does not look for a pricing function P. Instead, he *starts* from a given 'black box' pricing function P, say for instance the one that corresponds to a *fixed* local volatility function. Then he uses P to derive an expansion of the daily P&L, taking recalibration on the new market data into account, e.g. the fact that the local volatility is updated using the new market smile—thus describing and analyzing the inconsistent but common market practice. No dynamics of the market smile is provided or postulated; Bergomi simply expands the P&L at order two in the daily variations of the underlying price and implied volatilities (or option prices). If some gamma in the resulting P&L has no theta counterpart, Bergomi says that the 'model' (actually the routine of using P as the valuation function and recalibrating the market parameters every day) cannot be used for trading purposes.

Conversely, if each gamma in the carry P&L is compensated for by a payoff-independent theta, then Bergomi speaks of a *market model*: a 'model' that takes as inputs the prices of hedging instruments and does not systematically produce P&L leakage. The terminology 'model' is somewhat abusive though, as no dynamics of implied volatilities or option prices is provided. Bergomi's point is more to highlight a recipe for checking whether a market practice does make sense for risk management or not. In particular, he highlights that the local volatility model (recalibrated on a daily basis) is a 'market model' (Chapter 2), while the local stochastic volatility based on the Heston model (and recalibrated every day) is *not* (Chapter 12).

However, the downside of such an approach is that, when a 'market model' is identified, it provides no clue about why P should be considered a price. A replication argument is missing, which would require modelling the dynamics of the implied volatility surface. In fact, the only joint dynamics of the hedging instruments that is consistent with P is precisely that of the model that generated P in the first place, e.g. the model with fixed local volatility. Solving this inconsistency is tantamount to directly writing a joint model for an asset and its options...

Of course Bergomi goes beyond the local volatility model, presented as a particular case of stochastic volatility models. And after local volatility is studied at length (77 pages) in Chapter 2, Bergomi motivates, introduces and analyzes in detail several classes of stochastic volatility models, such as 'classical' one-factor stochastic volatility models like the Heston model, second-generation forward variance models, to which Bergomi's contributions are well known, local stochastic volatility models, as well as multi-asset stochastic volatility models.

The book unfolds as follows:

- Chapter 1 introduces 'market models', assesses the inefficiency of delta hedging, and motivates the need for and the use of stochastic volatility: by trading options for gamma hedging, one becomes sensitive to the future movements of implied volatilities, and stochastic volatility models will be a tool to model their dynamics.
- Chapter 2 is devoted to the simplest and most widely used stochastic volatilty model: the local volatility model. The classical links between option prices, local volatilities and implied volatilities are derived, as well as less known results about the joint dynamics of spot and implied volatilities in the model, and the vega-hedge. Then emphasis is put on the fact that it is a 'market model'—the simplest one, in fact. The uncertain volatility model is treated in the appendix as a particular case of (payoff-dependent) local volatility model.
- Chapter 3 introduces forward starting options, a specific class of options for which the volatility risk can be pinpointed. This case study allows Bergomi to showcase the drawbacks of the local volatility model and the benefits of using stochastic volatility models.
- Chapter 4, similar to Chapter 3 of the book *Nonlinear Option Pricing* by Guyon and Henry-Labordère, describes three attempts at directly modelling the joint dynamics of an asset and options written on it: direct modelling of the dynamics of implied volatilities of vanilla options; direct modelling of the dynamics of local volatilities; and direct modelling of the implied volatilities of power payoffs. All three attempts reach some kind of a dead-end, but allow the reader to better grasp where the difficulty lies, and motivate the use of forward variance models.
- Chapter 5 deals with variance swaps, their relationship to log contracts, and how they are impacted by large returns, dividends and interest rate volatility. Timer options are described and studied in an appendix, while the perturbation of implied volatilities around a lognormal density, at order one in the cumulants, is the subject of another appendix.
- Chapter 6 is devoted to the Heston model, expressed in terms of a forward variance curve model. Deficiencies of the Heston model are highlighted, in particular its inability to reproduce the observed spot/volatility

dynamics. Structural deficiencies of one-factor models are examined.

- The main course is Chapter 7, which devotes 90 pages to forward variance models, for which Bergomi is most known. These are stochastic volatility models that are built on the dynamics of instantaneous (continuous) forward variance swap variances and, by construction, are calibrated to an initial term structure of variance swap volatilities. Bergomi introduces a class of forward variance models that admit a Markov representation, and shows how the term structure of volatility of volatility, the term structure of the at-the-money-forward skew, and the smile of volatility of volatility can be controlled in such models. Discrete forward variance curve models are also examined; contrary to continuous forward variance models, they give independent handles on the vanilla smile, the smile of forward starting options, and volatilities of volatilities, which proves very useful when one risk-manages complex path-dependent options. Options on realized variance and VIX instruments are also covered.
- Chapter 8 (based on joint work with the author of this review) focuses on the smile of stochastic volatility models. An accurate expansion of the smile generated by general forward variance curve models, at order two in volatilities of volatilities, is derived, that characterizes the smile in terms of three dimensionless quantities that only depend on the spot/variance and variance/variance covariance functions.
- This characterization is used in Chapter 9 to establish a link between static and dynamic properties of stochastic volatility models. In particular, a link is established between the rate at which the at-the-money-forward skew decays with maturity and the skew-stickiness ratio, which quantifies how the at-the-money-forward volatility moves in average when the spot moves, in units of the at-the-money-forward skew.
- The last three chapters contain a lot of unpublished material. While Chapter 8 explored the links between a stochastic volatility model's specification and the smile it produces, Chapter 10 deals with actual equity smiles: what is responsible for their skew and curvature? Are vanilla smiles related to statistical properties of historical returns? Bergomi argues that the fat-tailed nature of the distribution of daily returns hardly impacts the at-the-money-forward skew, which is mostly produced by the covariance of spot and implied volatilities.
- In Chapter 11, Bergomi tackles the issue of multi-asset stochastic volatility modelling. This requires specifying not only spot/spot correlations, but also cross-asset spot/volatility and volatility/volatility correlations. Bergomi suggests a parametrization of the correlation matrix and derives approximate formulas for the at-the-money-forward basket skew, the basket variance swap volatility and the correlation swap.
- Finally Chapter 12 is devoted to local stochastic volatility models. After recalling how such models can be calibrated to the market smile using the particle method (or finite difference schemes for one-factor models), Bergomi explains why most local stochastic volatility models lead to P&L leakage and thus are not 'market models'. However, local stochastic volatility models based on lognormal stochastic volatility models are 'market models', and Bergomi provides estimates of the break-even gamma/theta levels that those models generate for the spot and implied volatilities of vanilla options, by deriving approximate formulas for the volatilities of volatilities, spot/volatility correlations, as well as for the skew-stickiness ratio.

On a formal note, the editorial quality and readability of the book is globally excellent. Each chapter concludes with a useful chapter's digest that recaps the main take-aways. The reader will particularly appreciate that many figures illustrate Bergomi's points. I have sometimes found the notation unclear, for instance when the author sometimes uses $\Sigma_i a_i$ for $\sum_i a_i$ (which could be misleading as Σ is usual notation for a volatility!), or when he uses brackets $\langle\rangle$ for both the quadratic variation (which is standard) and sometimes the expectation. Also, I have found that bibliographic references were sometimes missing. Bergomi maintains an errata webpage at http://www.lorenzobergomi.com/errata-contact.

To conclude, the modelling of the joint dynamics of assets and their options is still in its in infancy, and Bergomi's book enlightens this fascinating topic. In particular, asset prices and implied volatilities move as they wish (though they are not independent and are linked by strong no-arbitrage relationships), not according to a model, and Bergomi's starting point is to take this—annoying—reality into account. This is a difficult issue, so the answers are not completely neat, but Bergomi strives to convey his main ideas with great pedagogy. Stochastic volatility is introduced as a way to address issues arising in the modelling of derivatives, in particular to generate desired joint dynamics of an asset and its implied volatilities. Note that the book does not cover path-dependent volatility models, despite the fact that they too generate very rich joint spot/volatility dynamics. In particular, there is a strong, implicit assumption throughout the book that option prices depend on the past only through the *current* values of the hedging instruments.

Actually, with *Stochastic Volatility Modeling*, Bergomi has written the book that I wish I could have read when I joined the industry ten years ago!

With this book, Bergomi has actually offered a precious gift to the whole quant community: his very rich and concrete experience on volatility modelling organized in 500 pages and 12 chapters full of insights; and to the academic community as well: new ideas, points of view, and questions that could well feed their research for years.

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at Columbia University and at the Courant Institute of Mathematical Sciences, NYU. Before joining Bloomberg, Julien worked in the Global Markets Quantitative Research team at Société Gènèrale in Paris for six years (2006–2012), and was an adjunct professor at Université Paris 7 and Ecole des ponts. He co-authored the book *Nonlinear Option Pricing* (Chapman Hall, CRC Financial Mathematics Series, 2014) with Pierre Henry-Labordère. His main research interests include nonlinear option pricing, volatility and correlation modelling, and numerical probabilistic methods. Julien holds a PhD in Probability Theory and Statistics from Ecole des ponts. He graduated from Ecole Polytechnique (Paris), Université Paris 6, and Ecole des ponts. A big soccer fan, Julien has also developed a strong interest in sports analytics, and has published several articles on the FIFA World Cup, the UEFA Champions League, and the UEFA Euro in top-tier newspapers such as *The New York Times, Le Monde,* and *El País*, including a new, fairer draw method for the FIFA World Cup.